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University of Texas Bulletin
No. 2306: February 8, 1923

Physical &
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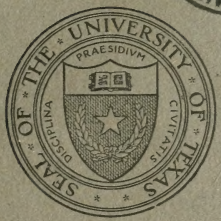
The Texas Mathematics Teachers' Bulletin

Volume VIII, No. 2



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Volume VIII, No. 2

PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH, AND ENTERED AS SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin
No. 2306: February 8, 1923

The Texas Mathematics Teachers' Bulletin

Volume VIII, No. 2

Edited by
MARY E. DECHERD
Instructor in Pure Mathematics,
and
JESSIE M. JACOBS
Instructor in Pure Mathematics

This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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MATHEMATICS FACULTY OF THE UNIVERSITY OF TEXAS

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THE CORRELATION OF THE TEACHING OF MATHEMATICS AND SCIENCE

In a private school it is possible to have a flexible program. If some topic introduces naturally some mathematical detail it is possible to take some time specially dealing with that kind of mathematics, there and then. In the high school this is made impossible by the rigid schedule of subjects hermetically sealed from one another. It is all the more important that the teachers in the various subjects meet together occasionally to see how far they can link up their work.

I do not suggest that the mathematical program be dominated or determined by teachers outside the department, but there should be a mutual search for topics and treatment, which may make the boy or girl realize that algebra applies to historical statistics, that geometry is helpful in geography.

If this mutual coöperation is impossible, it is yet possible for mathematics teachers to do something on their own initiative.

It is always a fresh source of astonishment that students who can handle a linear equation, for example, cannot apply it to problems of simple motion. I find it very difficult to divorce myself from the naïve belief that the student who knows a general method, will apply it easily whenever it is applicable. Whereas, actually, the difficulty of restating the problem in mathematical form is the greatest difficulty of all. It must be surmounted anew for each new case in which the same algebraic equation is applied. It is not to be wondered at that a student, who has had his algebra separated from all other work—even arithmetic and geometry—has a difficulty in making an application of his own accord.

Recently there has been some criticism of the necessity of mathematics to the boy or girl who is not going on to a university, and it is not difficult to believe that a large fraction of the algebra which is now taught is actually never

applied by the children in later life. This suggests that the field of mathematics should be broadened. On the other hand, by broadening the field I do not mean that mathematics be made a dollar-catching piece of machinery.

I am not urging that special attention be paid to solutions of particular technical problems which happen to arise in business or engineering. Mathematics is more than a numerical calculator. It is an attitude, a plan for grasping reality, for handling the environment by a consideration of its more general and abstract properties. Indeed, it is difficult to say exactly where mathematics stops and modern physics begins. In school it is, furthermore, the mathematics teacher rather than anyone else who can present to the student the broader view of science.

It is this scientific attitude, this striving for logic, for generality and precision, which is a crying need of the present time. If the children have grasped that, they have learned something which should belong to voting citizens of a democratic state, whether particular formulas are applied or not.

Now, this point of view cannot be presented by a conglomeration of high-sounding chatter. It can only be taught by the use of mathematical methods in the consideration of the simplest scientific topics. What I would plead for is a search by the teacher for those topics which are at the same time of cultural value, which can be handled with a fair degree of logical precision, and which are not too technical to be grasped by all who have reached the mental stage of the students in the actual class.

This cannot be achieved all in a moment, nor can it be done by the university professor alone. The experienced teachers must be the mainstay of such a movement—both because they know what can be effectively taught, and because they are in a position to make experiments.

With some diffidence I mention some examples to illustrate what I have been saying.

Let us take the example of indices. For most ordinary purposes any but the simplest integral indices are very little used. The general law of indices is usually presented

first, as an interesting study in the gradual expansion of a symbolism; second, as a step towards logarithms. But there is a natural source of indices in large and small numbers. The distance to the sun in feet, population, export and import statistics, areas of countries, etc., are best grasped by using symbols such as:

$$5\times10^{11} \text{ feet.}$$

These symbols are used frequently in physics but are unintelligible, at first, to students even when they have had some training in indices. Again, negative indices occur naturally in connection with the wave-length of light, the size of a molecule, etc.

In themselves large and small numbers always interest students (whether this interest is a legitimate one, intellectually speaking, or not). The whole of index theory should be developed first of all from this aspect, with 10 as a base. The student would see the value of a law which would enable us to multiply two large numbers by adding their indices. There is also the problem of finding the decimal point in division as, for example, in

$$.002\div.0003 \text{ or } .002\div.3.$$

Another topic, that of negative numbers, will be handled by Miss Red in a later paper. All I need to mention here is that the very essence of the negative number is its ability to express motion, or change in general. It can only be grasped in terms of motion and its study naturally leads to the study of directed motion. One need not teach the student mechanics, but surely they should learn to handle uniform north and south motion of a train along a track with negative as well as positive instants of time. If they cannot do that with their negative numbers it is no wonder that they rarely use algebra after leaving school.

Again in geometry, I believe it would force home the necessity of careful logic and the importance of the assumptions made to see what becomes of some important theorems on a spherical surface such as the earth. This presentation need not be at all rigorous, but specific examples will stimulate the students' critical faculties. We compare the plane triangle with the triangle of which one

vertex is the pole, the other two on the equator 90 degrees apart, a triangle all of whose angles are right angles. On the other hand the simple geometry is again true (at least, if we neglect Einstein, as we should for high-school students) for a triangle consisting of the earth, the sun and a star. Fortunately, at high school I was taught the rudiments (mere rudiments they were) of geometry on a sphere, and it opened my eyes to the meaning of plane geometry. How much could be done is a matter for the teacher to decide, but it would broaden both geometry and geography, particularly nowadays when the world is so small.

Again, there is a vast field of application for the equation $y=kx$.

Finally, I would plead for more and yet more work in graphs. In all subjects which make any pretensions of being scientific the graph is the most used mathematical tool. In itself it is the meeting point of arithmetic, algebra, geometry, and trigonometry, and illuminates all these branches of mathematics. It is practical and yet lends itself to logical precision. Last, but not least, through the graph can be presented the idea of change and history. Calculus cannot be taught at high school at present, but the general notion of a rate of change, of a distinction between increasing and decreasing can at least be studied. That an amount of money finally increases more rapidly at compound interest at however small a rate, than at simple interest, however great nominally, can be presented graphically. And the graph will be the only part of mathematics outside arithmetic which many high-school students will ever use in later life with conscious intent.

Lest I should be misunderstood, let me reiterate that my desire is not for teaching high-school students in such a way that they can handle college physics and chemistry more easily. Many of them never go to college. But they should be taught about such things that they may see how mathematics enables us to consider whole groups of problems, how it brings clarity into our conception of the environment. At the same time the topics chosen must not be

at random but so as to develop mastery in a few aspects of reality beyond the limited realm of pure numbers.

P. J. DANIELL,
Rice Institute, Houston, Texas.

THE SQUARE ROOT

It does not take a teacher long to find out that while there are children in his classes who try to refrain from exerting themselves mentally, there are others who have wide-awake and inquisitive minds and insist on knowing the "why" and the "wherefore." For the sake of these and in the hope of attaining the interest of those mentally inert, we teachers should not be content merely to present our subject matter, but in addition make that presentation so clear and broad that it can be readily understood.

When I was in the seventh grade I heard a pupil ask one of the teachers for assistance in solving a square root problem. The teacher replied that he could never remember the method of finding the square root and had to learn it every time he needed it. Impressed by the remark and thinking of the waste of time involved, I decided to learn to extract the square root so well that I would never have to relearn it. A proper presentation of the process involved should make it possible for pupils to retain their ability in this line for at least a year or two—let us hope for much longer. With the hope that it may aid some teacher, I am including my method of presenting evolution to classes in algebra.

To obtain the square root, use the formula $2t+u$ which means twice tens plus units. Lack of time really makes it impossible to give them the derivation of the formula.

The method employed follows. To find the square root of 529,984.

$$
\begin{array}{rr|l}
 & & 7 \\
\hline
 & & 52\ 99\ 84 \\
 & & 49 \\
\hline
2t = & 140 & 399 \\
u = & 2 & \\
\hline
\end{array}
$$

```
                    7  2
                 ----------
               | 52 99 84
               | 49
               ----------
2t = 140       |  399
 u =   2       |  284
-----------    ----------
2t + u = 142      11584
```

```
                       7  2  8
                    ----------
                  | 52 99 84
2t = 140          | 49
 u =   2          ----------
                    |3 99
                    |2 84
                    ----------
  2t = 1440         1 15 84
   u =    8        1 15 84
-------------       ----------
  2t + u = 1448
```

Algebra follows arithmetic. So correlate this with the square root in algebra. It is not necessary to add a "0" to the tens in algebra.

Thus:

```
                        3x²—9x+1
                    ----------------------------
                  | 9x⁴—54x³+87x²—18x+1
2t = 6x²          | 9x⁴
 u =     —9x      ----------------------------
                      —54x³+87x²—18x+1
2t + u = 6x²—9x     —54x³+81x²
                    ----------------------------
                                6x²—18x+1
2t = 6x²—18x                    6x²—18x+1
                                ----------
 u =        +1
-------------
2t + u = 6x²—18x+1
```

It is necessary to place the unit's expression to the side of 2t and not immediately under, because in more involved problems, it is necessary to place x^3, x^2, etc., under each other to add.

Then going to the cube root and again taking the arithmetic problem first, we have the rule: $3t^2+3tu+u^2$.

Cube root:

$3t^2=300$
$3tu=\ 30$
$u^2=\ 1$
$3t^2+3tu+u^2=331$

1 | $1\ 404\ 928$
1

404
$1\quad 1\quad 2$

$1\ 404\ 928$
1

404
331

$73\ 928$
$73\ 928$

$3t^2=300$
$3tu=\ 30$
$u^2=\ 1$
$3t^2+3tu+u^2=331$

$3t^2=36300$
$3tu=\ 660$
$u^2=\ 4$
$3t^2+3tu+u^2=36964$

Then following this in algebra we have:

$3a^2-a+1$

$27a^6-27a^5-18a^4+17a^3+6a^2-3a+1$
$27a^6$

$3t^2=27a^4$
$3tu=\ -9a^3$
$u^2=\ a^2$
$3t^2+3tu+u^2=27a^4-9a^3+a^2$

$-27a^5-18a^4+17a^3+6a^2-3a+1$
$-27a^5+\ 9a^4-\ a^3$

$-27a^4+18a^3+6a^2-3a+1$
$-27a^4+18a^3+6a^2-3a+1$

$3t^2=27a^4-18a^3+3a^2$
$3tu=\ -3a^2-3a$
$u^2=\ +1$
$3t^2+3tu+u^2=27a^4-18a^3+6a^2-3a+1$

The pupils will find it very interesting to work their fourth and sixth root problems by the formula:

$$4t^3+6t^2u+4tu^2+u^3 \text{ and}$$
$$6t^5+15t^4u+20t^3u^2+15t^2u^3+6tu^4+u^5,$$

respectively, as well as by obtaining the square root of the square root and the square root of the cube root. The teacher can readily work out the formulas for the fifth root, the seventh root, etc.

In all our teaching and dealing with our pupils, let us

remember, "Unto him that hath . . ." We are at least supposed to have, so let us *give* bountifully.

JESSIE MARIE FOUTS.

Although not explicitly stated, Miss Fouts bases the extraction of all roots on the binomial formula $(x+y)^n$, n integral. Evidently the use of the theorem for $n>3$ is to be applied after studying this rule in algebra.—Editors.

THE AUTHORS OF THE WENTWORTH-SMITH GEOMETRY—BRIEF BIOGRAPHICAL SKETCHES

Readers of the *Mathematics Teachers' Bulletin* may be interested in learning a few facts concerning the lives of the authors of the state textbook in plane geometry, so that the following brief sketches are given herewith.

George A. Wentworth was born in Wakefield, Carroll County, New Hampshire, on the 31st of July, 1835. He was the son of Edmond and Elisa (Lang) Wentworth and a descendant of a William Wentworth, who emigrated from England to Massachusetts in 1638, and who was one of the founders of the town of Exeter and was later a resident of Dover, New Hampshire. As a boy, George Wentworth attended the academy in Wakefield for about one year. He then went to Phillips Exeter Academy. He left this academy in 1855 to enter the sophomore class at Harvard College, from which he graduated in 1858. Immediately on graduation he was invited to return to Phillips Exeter Academy as instructor in ancient languages. In the following year he was appointed head of the department of mathematics, and remained in this position for thirty-two years. In 1891 he left his academic work to become president of the Exeter Banking Company, with which he continued until shortly before his death in 1906. He was married at Covington, Kentucky, on the 2nd of August, 1864, to Emily J. Hatch, the daughter of Daniel G. and Mary R. Hatch. At his death he left two sons and one daughter. George Wentworth is one of the few mathematics teachers of the country to become wealthy as a result of his textbooks. These proved exceedingly popular and have gone through many editions and revisions. The following dates of first editions may be mentioned: *Elements of Algebra,* 1878; *Elements of Geometry,* 1881; *Plane and Spherical Trigonometry,* 1882; *Surveying and Navigation,* 1882; *Five Place Table of Logarithms,* 1882; *Elements of Analytic Geometry,* 1886; *School Algebra,* 1887; *College Algebra,* 1888; *Higher Algebra,* 1891. Dr. Wentworth was also au-

thor of several arithmetic books and joint author of a book on physics.

David Eugene Smith was born in Cortland, New York, January 21, 1860. He is the son of the Honorable Abram P. and Mary E. (Bronson) Smith. He graduated from Syracuse University in 1881 with the degree of Bachelor of Philosophy. Three years later he secured there a degree of Master of Philosophy, and in another three years a degree of Doctor of Philosophy. For the first three years after graduation he practiced law in his native town. After receiving his master's degree he was appointed to the mathematics department of the State Normal School in Cortland, and continued there in this position until 1891. For the next seven years he was professor of mathematics at the Michigan State Normal College, receiving the degree of Master of Pedagogics at the close of this period. From 1898 to 1901, he was principal of the New York State Normal School at Brockport and was appointed in 1901, professor of mathematics in the Teachers College, Columbia University. In 1905 he received the degree of LL.D. from Syracuse University, his Alma Mater. At various periods he has been in Europe engaged in study and research. Professor Smith is today head of the mathematics department of the Teachers College, Columbia University, and is in a position which exerts considerable influence upon the trend of elementary teaching in this country.

Professor Smith is a frequent contributor to the mathematical journals of this country and abroad, usually on topics of elementary teaching of history and mathematics. Among his published books may be mentioned the following: *History of Modern Mathematics,* 1896; *Teaching of Elementary Mathematics,* 1900; *Primary Arithmetic,* 1904; *Grammar School Arithmetic,* 1904; *Algebra for Beginners,* 1904; *Intermediate Arithmetic,* 1905; *Advanced Arithmetic,* 1905; *Practical Arithmetic,* 1905; *Outlook for Arithmetic in America,* 1905; *Rara Arithmetica,* 1907; *Teaching of Arithmetic,* 1909 (this was revised and enlarged in 1913);

Teaching of Geometry, 1911; *Topical Primary Arithmetic*, 1912; *Topical Practical Arithmetic*, 1912; *History of Japanese Arithmetic*, 1912; *Number Stories of Long Ago*, 1915; *the Sumario Compendioso of Juan Diez*, 1920; *Our Indebtedness to Greece and Rome in Mathematics*, 1922.

ALBERT A. BENNETT.

HOMOTHETIC FIGURES

If the vertices of a polygon are joined to a point in its plane and if on these lines segments are laid off from the point so that their ratios to the joining lines are all equal to each other, then the outer extremities of these segments are the vertices of a polygon similar to the given polygon and the corresponding sides of the poloygons are respectively parallel.

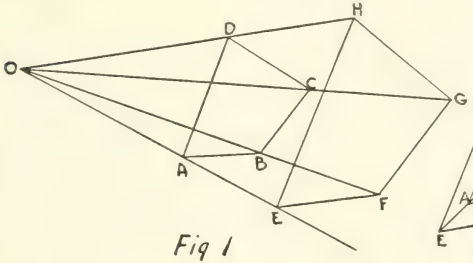


Fig 1

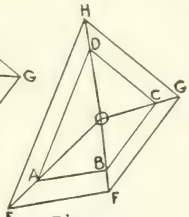


Fig 2

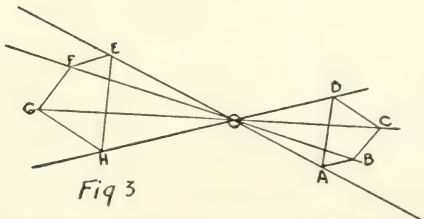


Fig 3

Let A B C D be the given polygon and O the given point (Figs. 1, 2, 3). Join O with the vertices A, B, C, D and on OA, OB, OC, OD lay off the segments OE, OF, OG, OH, respectively, so that

$$\frac{OE}{OA} = \frac{OF}{OB} = \frac{OG}{OC} = \frac{OH}{OD}$$

Then E F G H is a polygon similar to the poloygon A B C D and E F is parallel to A B.

F G is parallel to B C, G H is parallel to C D and H E is parallel to D A.

The triangles O E F, O A B have by hypothesis $\frac{OE}{OA} = \frac{OF}{OB}$ and the angle E O F=angle A O B, therefore

$$\frac{EF}{AB} = \frac{OF}{OB}.$$

In like manner it is proved that the triangles OFG, OBC are similar, therefore $\frac{FG}{BC} = \frac{OF}{OB}$.

Hence $\frac{EF}{AB} = \frac{FG}{BC}$

that is any two adjacent sides are proportional.

Since the triangles OEF, OFG, etc., are respectively similar to the triangles OAB, OBC, etc., the polygons are equiangular and hence similar and the corresponding sides of the polygons are parallel.

Definition. Two similar polygons whose corresponding sides are parallel to each other are said to be homotheic. In Figures 1 and 2 the polygons are *directly* homothetic. In Figure 3 the polygons are *inversely* homothetic. The point O is called the homothetic center and the constant ratios OE: OA or EF:AB is called the homothetic ratio. Any line through O, the homothetic center, meeting the perimeters of the figures intersects the polygons in two *corresponding* points the ratio of the segments of the line determined by the perimeters and the homothetic center being equal to the homothetic ratio i.e., if x and y are the points of intersection of the line with the polygons E F G H, A B C D, OX:OY=OE:OA.

2. If two similar polygons have their sides respectively parallel the lines joining corresponding vertices meet in a point.

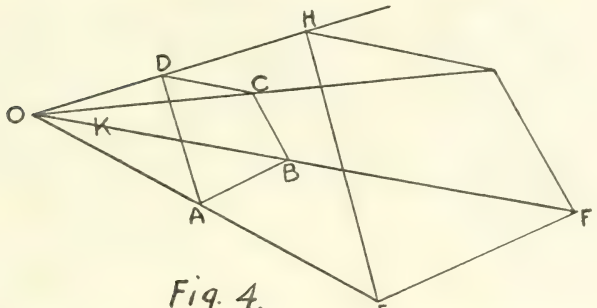


Fig. 4.

Let the polygon EFGH be similar to the polygon ABCD and let EF, FG, etc. be respectively parallel to AB, BC, etc., then EA, FB, GC, HD meet in a point (See Fig. 4).

Let EA and FB meet in O and GC and FB in K. Then

OE:OA=OF:OB=EF:AB

In the triangles FKG, BKC

FK:BK=GK:CK=FG:BC.

By hypothesis EF:AB=FG:BC,

therefore FO:BO=FK:BK,

therefore FO—BO:BO=FK—BK:BK

i.e., BF:BO=BF:BK

therefore BO=BK

and since K and O are on the same side of the point B, therefore K coincides with O i.e., the lines EA, FB, GC meet in a point. In like manner it is proved that HD passes through O.

3. Given a fixed point O and a variable point E, if the line OE is divided by the point A in a constant ratio then if point E traces out a polygon EFGH, the point A will trace out a polygon similar to the polygon EFGH, the homothetic center being O and the homothetic ratio being the constant OE:OA.

Let EFGH, Figure 4, be the polygon traced by E. Join OF and determine the point B corresponding to F from the proportion

OE:OA=OF:OB.

Hence B lies on a parallel through the point A to the line EF. As E moves on any line, A moves on a line parallel to that line, and the distances OE, OF, OG, OH, are divided in the constant ratio OE:OA, hence by proposition one above the polygons are homothetic.

4. If O is a fixed point and E a variable point which traces out a circle and if OE is divided in the point A so that OE:OA, remains always constant, then the locus of A is a circle.

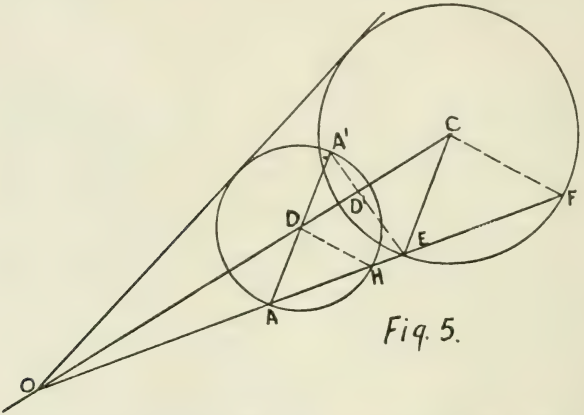


Fig. 5.

Let C be the center of the circle traced out by E (Fig. 5). Join the fixed point O with C and find the point D on OC which satisfies the condition OE:OA=OC:OD.

Then D is a fixed point since it divides the segment joining two fixed points O and C in a ratio which is constant.

In the triangles OEC, OAD,

OE:OA=OC:OD and angle AOD is common, hence the triangles are similar, therefore AD:EC=OA:OE

$$\text{therefore } AD = \frac{OA}{OE}. \quad EC$$

The ratio $\frac{OA}{OE}$ is a constant since it is the reciprocal of a constant ratio, CE is constant since it is the radius of a

given circle hence AD is constant i.e. A moves at a constant distance from a fixed point D and describes a circle whose center is D.

Cor. 1. If OE meets the given circle again in F and the circle D in H and if F is joined with C and H with D the triangles AHD, ECF being isosceles and angle A equals angle E, therefore angle H equals angle F, hence DH is parallel to CF and consequently OF:OH=OC:OD=OE:OA.

Hence the important conclusion, the line joining the extremities of parallel radii drawn on the same side of the line of centers of two circles passes through their homothetic center.

Cor. 2. The centers of two circles are corresponding points.

Cor. 3. The extremities of parallel radii drawn in the same direction are corresponding points.

Cor. 4. The external common tangent of two circles divides the line of centers of the circles in the ratio of the radii of the circles. Hence the external common tangent of two circles passes through the homothetic center or center of similitude of the two circles.

Cor. 5. If DA′ is drawn parallel to CE and A′ is joined to E, CD, the line of centers is divided by A′E in D′ in the ratio of the radii. The point D′ is also a homothetic center of the circles. As D′E describes the circle C, D′A′ will describe the circle D but in reverse order, i.e. as E moves over the part of the circle C on one side of the line of centers A′ will move over the part of the circle D on the other side of the line of centers.

Cor. 6. If two circles are tangent externally the point of contact is a homothetic center.

PROBLEMS

1. In a given triangle inscribe an equilateral triangle having its vertices on the sides of the given triangle.

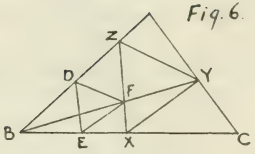


Fig. 6.

Let the given triangle be ABC (Fig. 6). Draw a line DE and on it describe an equilateral triangle DEF. Join BF and let BF meet AC in Y. Draw YX parallel to FE, YZ parallel to FD. Join XZ. The required triangle is XYZ.

BX:BE=BY:BF=BZ:BD.

Hence by proposition 1, XYZ is homothetic to EDF.

Remark: As DE may be drawn arbitrarily the number of solutions is infinite.

2. Inscribe a square in a triangle.

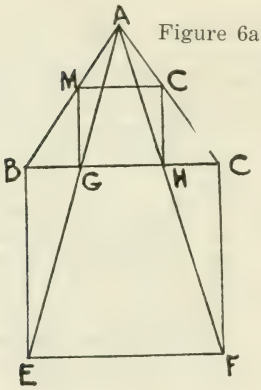


Figure 6a

Let the triangle be ABC. On BC describe a square BEFC. Join AE, AF and let G and H be their points of intersection with BC. Draw GM, HL perpendiculars to BC intersecting AB, AC in the points M, L.

GHLM is the required square.

For AM:AB=AG:AE=AH:AF=AL:AC.

Hence the figures are homothetic.

Remark: There is only one solution.

3. In a given sector of a circle inscribe a square.

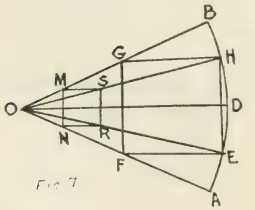


Fig. 7

Let AOB be the sector (Fig. 7). Bisect angle AOB by the straight line OD. Take any point M on OB and draw MN perpendicular to OD. On MN describe a square MNRS. Join OS, OR and let OS, OR meet the arc AB in H and E. Through E and H draw parallels EF, HG to Rn, SM. HGFE is the

required square.

For OF:ON=OE:OR

and OH:OS=OG:OM

OE=OH, OR=OS

therefore OF:ON=OE:OR=OH:OS=OG:OM.

Hence HGFE is homothetic to SMNR.

4. Draw a polygon similar to a given polygon ABCDE, so that the diagonal corresponding to BD may be equal to a given line.

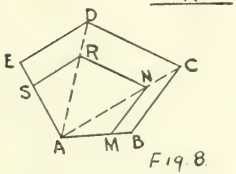


Fig 8

Find the fourth proportional to BD (Fig. 8), K, the given line, and AB, and lay it off on AB. Let AM be this line. Draw the diagonals AC, AD and draw MN parallel to BC, NR parallel to DC, RS parallel to DE.

AMNRS is the required polygon.

For, AM:AB=AN:AC=AR:AD=AS:AE.

Hence the polygons are homothetic.

Also, since AM:AB=AR:AD, triangles AMR, ABD are similar and hence BD:MR=AB:AM, but by construction BD:K=AB:AM

therefore MR=K.

5. Given two circles and a point A in their plane, to draw through the point A a straight line meeting the circles at B and C, so that AB:AC=m:n (Wentworth's Geometry, page 179).

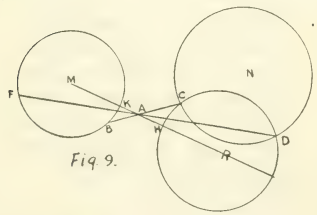


Fig 9.

Let the circles be M and N (Fig. 9), and A the given point. Join A with M and find the fourth proportional to m, n, AM and lay off on AM, AR equal to this fourth proportional. To m, n and the radius of the circle M, find the fourth proportional and with R as center and this line as radius describe a circle R intersecting the circle

N in C, D. Join CA, DA and produce CA, DA to meet the M in B and F. The required solutions are BC, FD.

Let K and H be the points of intersection of the line MR with the circles M and R. As K describes the circle M, H describes the circle R the point A being the homothetic center; and the homothetic ratio being n:m. C and B are corresponding points on the circles R and M, so also are D and F corresponding points on the circles R and M.

For, MA:AR=m:n.

MK:HR=m:n

therefore MA:MK=AR:HR

therefore MA—MK:MK=AR—HR:HR

i.e. KA:MK=AH:HR

therefore KA:AH=MK:HR=m:n.

therefore AH:KA=HR:MK=n:m.

Triangles MAB, RAC are similar

therefore AB:AC=MA:AR=m:n and triangles MAF, DAR are similar

therefore FA:AD=MA:AR=m:n.

6. Through a given point P within a circle to draw a chord AB, so that AP:PB shall equal the ratio m:n (Wentworth and Smith's Geometry).

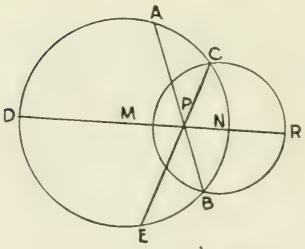


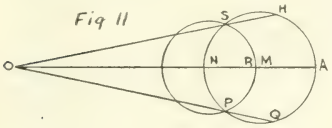
Fig 10.

Let ABD (Fig. 10) be the given circle and P the given point. Draw the line MPN through the center of the given circle. Find the fourth proportional to m, n, PD and lay off on MPN, PR equal to the fourth proportional. As PD describes the circle M, PR describes a circle N homothetic to it, the homothetic center being P and the homothetic ratio being n:m. Let the circles intersect in B and C. Join BP, CP. The solutions are AB, CE.

To get the center and radius of the circle N lay off from

R, RN the fourth proportional to m, n and the radius of the circle M.

7. Through a point O draw a line cutting a circle in P and Q so that the chord PQ is 1 5 OQ. (Godfrey and Seddon's Modern Geometry).



Let the circle M be the given circle and O the given point (Fig. 11). Draw OA through the center M. Lay off from A, a segment AR equal to 1/5 OA and make RN equal 4 5 AM. With N as center and NR as radius describe a circle intersecting the circle M in P and S. The required line is OPQ or OSH.

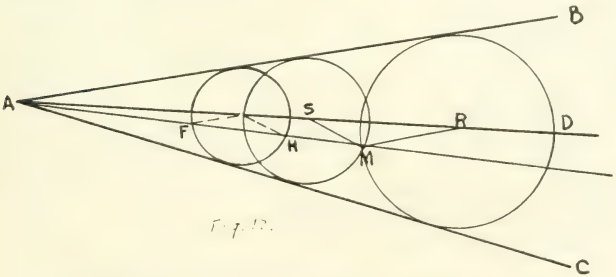
For OR:OA=4:5

NR:MA=4:5.

As A describes the circle M, R describes the circle N, hence OP:OQ=4:5.

i.e. PQ=1/5 OQ.

8. Construct a circle to touch two given lines and pass through a given point.



Let the lines be AB, AC and the point be M.

Take any point E on the bisector AD of angle CAB (Fig. 12). With E as center and the distance of E from AB as radius describe a circle tangent to AB, AC. Join AM and let the points of intersection of AM with the circle be F and H. As F describes the circle E with A as homo-

thetic center, the point M will describe the homothetic circle of the circle E. To determine the center draw MR parallel to FE. The circle with R as center and MR as radius will be a solution.

A second solution is obtained by considering H and M as corresponding points. Draw MS parallel to HE and describe a circle with S as center and SM as radius. The circle S is homothetic to the circle E.

There are two and only two solutions to the problem.

9. Given a circle and two radii OA, OB. Draw a tangent to the arc AB so that the point of contact will divide the segment of the tangent intercepted by the radii OA, OB in any ratio m:n.

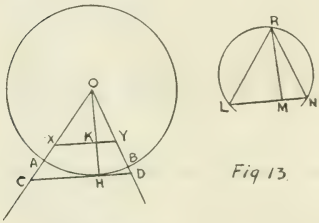


Fig 13

Let O (Fig. 13) be the given circle and OA, OB be the given radii, it is required to draw a tangent to the arc AB so that CH:HD=m:n.

Take two segments LM, LN in the same straight line so that

LM:MN=m:n

On LN describe a segment of a circle containing an angle equal to angle AOB. At M erect a perpendicular MR to LN. Join LR, NR. Then angle LRN equals angle AOB. On OA take any point X and make angle OXY equal to angle L. Draw OK perpendicular to XY and let it intersect the arc AB in H. Through H draw CD parallel to XY. CD is the required tangent. The triangle OCD is homothetic to OXY and XK:KY=m:n.

Hence CH:HD=m:n.

P. H. Underwood.

DESCRIPTIVE GEOMETRY

Descriptive geometry is the geometry of form and position in space, treated by representing objects in space by means of drawings made on one or more planes. These representations are called projections and the planes on which they are made are called planes of projection. The projections, because of their definite relation to the objects represented by them, are exact graphical descriptions of these objects. Hence, the name *descriptive* geometry.

Descriptive geometry deals first with methods of representing the objects of space by means of drawings in the plane; second, with the solution of space constructions by means of plane constructions; and third, with the demonstration of the above processes.

In the Mongean Method of Double Orthographic Projection a point P is projected orthographically upon each of two perpendicular planes. These planes, which are generally supposed to be horizontal and vertical, are called, respectively, the horizontal plane π_1 and the vertical plane π_2 (Fig. 1). The planes, π_1 and π_2, intersect in a straight line called the ground line, g_{12}. The orthographic projections

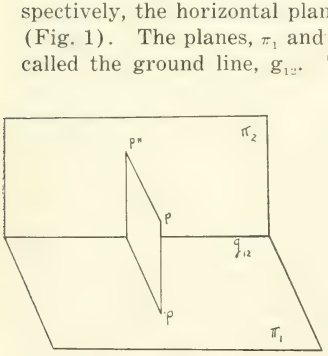


Fig 1

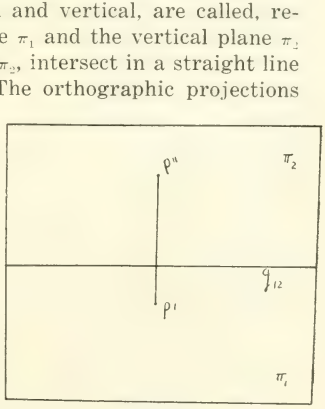


Fig 2.

of P up π_1 and π_2 are denoted by 'P and P'', respectively. In solving problems, it is not practical to have a horizontal plane and a vertical plane, and it is, therefore, customary

to assume that the plane π_1 is rotated around the ground line into coincidence with π_2 so that 'P then assumes the position P' (Fig. 2).

The plane π_2, with which the revolved position of π_1 now coincides, serves as a drawing plane and is called the Mongean plane in honor of Gaspard Monge (1746-1818) the French mathematician, who developed descriptive geometry as an academic study. To the point P of space there corresponds in the drawing plane the two points P' and P'' which lie on the same perpendicular to the ground line, and, conversely to such a pair of points in the drawing plane there corresponds but one point in space. Similarly, to a line, l, of space there corresponds a pair of lines l' and l'' in the drawings plane (Fig. 3), and, conversely to any pair of lines in the drawing plane there corresponds, in general, but one line in space. Instead of being determined by its projections l' and l'', a line l may also be given by its traces P_1 and P_2, that is, by the points in which it pierces π_1 and π_2, respectively. A plane of space is represented by its traces, its lines of intersection with π_1 and π_2 respectively, hence we can say that to a plane α of space there corresponds in the drawing plane a pair of lines a_1 and a_2 which intersect on the ground line (Fig. 3), and, conversely to such a pair of lines of the drawing plane there corresponds but one plane of space.

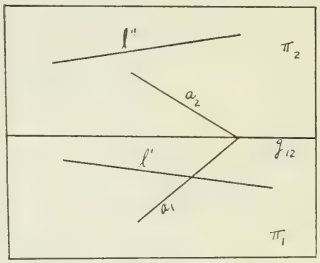


Fig 3

The problems of descriptive geometry are: Problems of position which involve only location; perpendicularity problems, and metrical problems, which deal with measurements of length and angle. One of the simple problems of position is: Given the vertical projection of a point P which lies on a given line, to find the horizontal projection of P.

Given P″ and l=[l′, l″] (P lies on l) (Fig. 4).

To find P′.

Through P″ draw a line perpendicular to g_{12} and produce it to meet l′ in the required point, P′. P′, since it must lie on the perpendicular to g_{12} through P″ and also on l′, is the

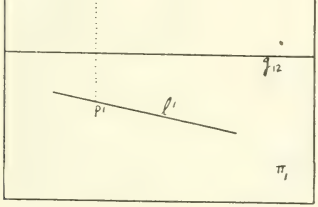


Fig 4

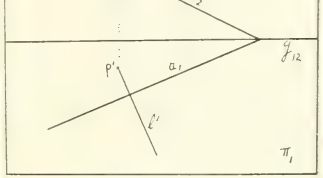


Fig 5

point of intersection of the perpendicular and l′.

One of the simple perpendicularity problems is: Through a given point P, to draw a line perpendicular to a given plane ∝.

Given P≡[P′, P″] and ∝≡[a_1, a_2] (Fig. 5).

To draw a line through P perpendicular to ∝.

Through P′ and P″ draw l′ and l″ perpendicular to a_1 and a_2, respectively. Then l passes through P by construction and is perpendicular to ∝ since the projection of l are perpendicular to the traces of ∝.

One of the simple metrical problems is: Given two points, P and R, to find the true length of the line segment between the two given points. Given P=[P′, P″] and R=[R′, R″]. To find the true length of PR pass a plane, ∝ (Fig. 6) through the lines P′P″ and R′R″. Rotate ∝ on P′P″ as an axis until it is parallel to π_2, the vertical plane of projection. Then P′R′ will take the position m′ parallel to g_{12}. RS is parallel to π_1, the horizontal plane of projection, by construction. Therefore R″S″, the vertical projection of RS will be parallel to g_{12}. Then we know S″, the intersection

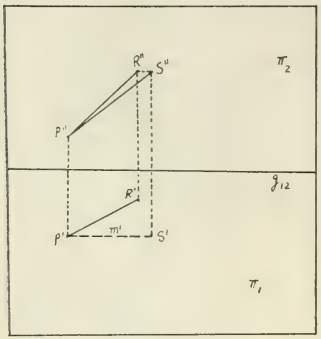


Fig. 6.

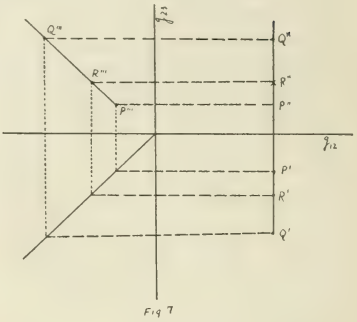


Fig. 7

of R″S″ and S′S″. Then P″S″ is the true length of the line segement between P and R, because after revolving PR to a new position parallel to π_2, its projection on π_2 is equal to its true length.

Simple solutions of some of the problems of descriptive geometry are given by means of the auxiliary profile plane π_3, a plane perpendicular to each of the planes π_1 and π_2 π_3 is revolved about g_{23}, the line of intersection of π_2 and π_3, as an axis until it coincides with the drawing plane and the profile projection of P, l, and the profile trace of $\propto$ are denoted by P″′, y″′, and a_3, respectively. Suppose we wish to solve a simple problem by means of the profile plane: Given a line perpendicular to g_{12} and the horizontal projection of a point on it. Find the vertical projection. Let P′P″ and Q′Q″ (Fig. 7) represent a line perpendicular to g_{12}. Given R′, the horizontal projection of a point on this line. Required R″, the vertical projection. Locate P″′ and Q″′. These two points determine the profile projection P″′ Q″′ of PQ. Locate R″′ on this line. Then a line through R″′ perpendicular to g_{23} will meet P″Q″ in R″, the required point.

Since orthographic projections do not always give an adequate picture of an object, an axonometric projection is frequently used for illustrative purposes. An axonometric projection of an object may be obtained by drawing

the orthographic projection of that object on a plane inclined to the principal axes of the object. These projections are generally constructed by setting off, by means of axonometric axes and scales, the rectangular coordinates of a sufficient number of points to represent the object clearly. To prepare an axonometric projection, it is, therefore, necessary to have the projections of the coordinate axes to which the principal lines of the object are parallel and the axonometric scales by which the coordinates of the points can be set off along the axes. The most common type of axonometric projection is the isometric where the three angles between the axes each equal 120° (Fig. 8). The most interesting thing in axonometric projection, I think, is to see a set of points grow into the picture of some familiar object. For instance suppose we are given the Mongean representation of a box (Fig. 9). It is only a few lines while the axonometric representation *is* the box (Fig. 10). The architect shows the blue prints to his manager but an axonometric representation is shown to the prospective buyer.

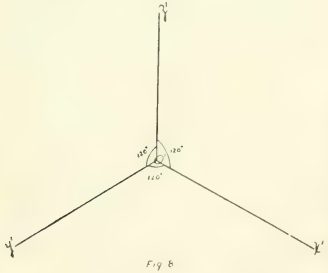


Fig 8

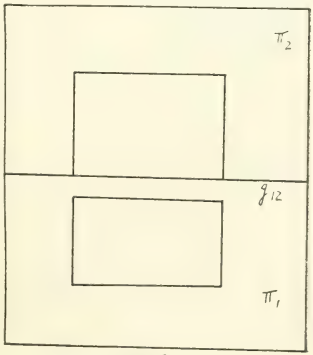


Fig. 9

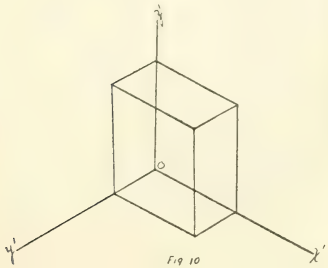


Fig 10

In *Free Perspective* a line of space is represented by two points in the picture plane, its trace and its vanishing point, a plane of space is represented by its trace and its vanishing line and a point is represented by its projection and the representative of a line or a plane on which it lies.

In the Method of Contour Lines a point is represented by its orthographic projection on a drawing plane and the number which represents its distance from this plane, a line is represented by its projection and a scale on this projection, and a plane is represented by one of its lines of greatest slope.

These several methods of descriptive geometry are used extensively in solving problems of mechanics and the constructive arts, for example in the planning of machinery and arches; problems concerning fortifications, as representing the natural surface of the ground in topographical and hydrographical maps; and problems of architecture. Because the representation of space objects by drawings in a plane is aided by representations of the shadows which they cast upon themselves and upon other objects as well as the shading, that is the reproduction of the intensity of illumination of the different parts of the surface of the body represented, the theories of shadows and illumination have become a branch of descriptive geometry.

Photogrammetry, a comparatively new branch of descriptive geometry, was first tested in 1851 by the French mathematician, Laussedat. Photogrammetry has for its aim, as the name implies, the obtaining from photographs correct metrical representations of the object photographed. In supplying a means of obtaining drawings of enemy fortifications from a few hastily taken snapshots photogrammetry makes a decided contribution to military science. With skillful aviators who take snapshots, not only of fortifications but of factories, manufacturing plants and other important buildings back of the enemies' lines, photogrammetry is becoming increasingly important. It was used extensively during the World War. It also finds application in certain kinds of research work. For instance, the only means by which it is reasonably possible to secure

accurate measurements of clouds, lightning, the northern light, stars, glaciers, or sea waves is by direct photography followed by the application of the methods of photogrammetry.

Photogrammetry is making a rich contribution to the investigation of historic architecture and paintings. In the Royal Institute in Berlin, there are more than fourteen thousand pictures of the buildings of Germany, as well as a few pictures of buildings of other countries, which by furnishing the structure of buildings for a long period of time, make it possible for the art of photogrammetry to materially aid the art of architecture.

Surgery is another art which is profiting by the contributions of photogrammetry. X-ray pictures of a bullet or other foreign object lodged in the body, or of diseased organs of the body, followed by the application of the methods of photogrammetry, give information to the surgeon which is of great importance.

In 1889, E. Deville, by the methods of photogrammetry, surveyed the western provinces of the Dominion of Canada. In 1922 the Forest Service of the United States Department of Agriculture used airplanes for locating and photographing undiscovered lakes in the national forests of Alaska. Recognizing that many of these lost lakes may be sources of valuable water power, the Forest Service has laid plans to map this land of the north by means of aerial photographs and the methods of photogrammetry. A few days' flight, it is said, will be sufficient to cover the area with a degree of accuracy that would require many years and great expense to accomplish by ordinary methods. Thus, photogrammetry has, by its contributions to the practical arts, justified the study of higher geometry from a purely utilitarian standpoint.

On account of these various branches of descriptive geometry, it is clear that the technical schools should devote a great deal of time to the teaching of this subject, although the teaching should not be confined to technical schools. Descriptive geometry enables the student to visualize space forms. The inability to visualize space forms

interferes not only with his study of other branches of geometry, but with his study of sciences where very frequently a mental image of a situation is as important as, or leads to the analysis of the situation. William H. Roever, an outstanding man in advancing the study of descriptive geometry in this country, points out the fact that "a general course in mathematics without graphical solutions of plane and space problems is almost as great an incongruity as general courses in physics and chemistry without laboratory work."

The high-school teacher of mathematics will find the study of the graphical solutions of plane and space problems helpful in presenting plane and solid geometry; the college teacher of mathematics will find in the study an opportunity to advance a branch of mathematics which is becoming increasingly important; while all teachers of mathematics and teachers of other subjects will find the study interesting and stimulating, so I say to you: Study Descriptive Geometry.

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